FILTERED DOPPLER VELOCIMETER: DEVELOPMENT OF A POINT SYSTEM

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Abstract

A velocity measurement device has been developed which uses a Distributed Brag Reflector (DBR) diode laser and a Cesium vapor Faraday cell to measure the Doppler shift of scattered light. The device, referred to as the Point Doppler Velocimeter (PGV), combines many of the advantages of laser Velocimetry with the advantages hot wire anemometry. Light from a single frequency DBR diode laser is focused on a seeded flow field. Doppler shifted light scattered by the seed particles is collected and passed through a Cesium vapor Faraday cell which acts as the frequency discriminator for the system. The Faraday cell provides a linear frequency dependent transmission edge which converts changes in the frequency of the Doppler shifted light to changes in signal intensity. The resulting system makes non-intrusive velocity measurements with directional sensitivity comparable to a laser velocimeter. The system also provides a continuous velocity signal similar to a hot wire anemometer. Results that demonstrate the accuracy and bandwidth of the system are reported. The device has been used to make velocity measurements on a rotating disk and in a jet.

Introduction

The velocity field is among the most common quantities of interest in many fluid mechanics experiments. Current techniques for measuring fluid velocity include the pitot-static tube, hot wire anemometer, and laser velocimetry. Goldstein[1] set forth a set of criteria for the ideal velocity measurement instrument, several of these criteria are:

(1) High frequency response.
(2) Small probe volume.
(3) High accuracy and resolution.
(4) Sensitive only to velocity.
(5) Measure velocity components and detect flow reversal.
(6) Create minimal flow disturbance.

Both laser velocimetry and hot wire anemometry meet many of these criteria, however, each in some aspects. Hot wire anemometry combines high frequency response with good accuracy and resolution. However, the hot wire is sensitive to density and temperature as well as velocity and it cannot detect flow reversal. The laser velocimeter is sensitive only to a single component of velocity and can be modified to detect flow reversal. The frequency response of current laser velocimeter systems is somewhat lower than the hot wire anemometer at about 30 kHz. An instrument, which could combine the frequency response of the hot wire anemometer with the directional sensitivity of the laser velocimeter, would prove to be a useful tool for many studies of fluid mechanics.

In the past decade a new laser velocimetry technique known as Doppler Global Velocimetry (DGV) has been under development. DGV uses a laser to scatter light off of a heavily seeded flow. The Doppler shift of the scattered light is determined using a molecular absorption filter as the frequency discriminator. The molecular filter has an absorption edge that is frequency dependent; thus a change in frequency of the scattered light is converted to a change in signal intensity at the detector. In the Global scheme, a CCD camera is used to image a section of the flow field producing a two dimensional grid of velocity data. A point scheme, the point Doppler velocimeter (PDV), is constructed by replacing the CCD camera with a photodiode or photo-multiplier tube. The system shares many of the characteristics of other laser velocimetry techniques; it is sensitive only to the velocity of the scattering particles. The frequency response of a PDDV system should be limited only by the photo detector used to image the flow. In the case of a point system which uses a photodiode or photo-multiplier tube this frequency response is potentially very high.

Background

Komine[2] first suggested the concept of using a molecular absorption filter as the frequency discriminator in a laser velocity measurement system. Since that time researchers have developed both global and point schemes using various combinations of

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molecular and atomic filters and lasers. Global systems employing molecular iodine and either a pulsed Nd:YAG or Argon Ion laser have been developed by Meyers[3], Komine[2], McKenzie[4], and Beutner[5]. Results have been reported in several high and low speed flows. McKinzie suggest a minimum possible velocity resolution of 2 [m/s] for a global system based on current techniques, the frequency stability of lasers as well as the CCD camera characteristics are considered the to be the major sources of error.

Point systems have been developed by Hoffenberg[6], Bloom[7] and Menders[8 9 10], Kuhlman[11], and Miles[12]. Hoffenberg developed a point system using an Ar+ laser and a molecular iodine cell. The point system was applied to a turbulent jet. Mean velocity measurements in a turbulent jet agreed within 2% when compared to standard LV measurements.

Several variations of a velocity measurement system which use a Faraday cell as the frequency discriminator have been developed by Menders and Bloom. Early systems focused on the measurement of mean velocity. Recently, velocity measurements of a rotating disk are reported for a system in which stop band of the Faraday filter is modulated about the laser operating frequency. These results indicate an accuracy of better than 5 [cm/s] when operating at a 30 ms integration time.

Kuhlman has developed a 2 component PDV system using an Ar^+ laser and a molecular iodine cell. The system was used to make velocity measurements on a rotating disk; results indicate an average error of about 0.7 [m/s].

Miles measured fluid velocity by locking the Rayleigh scattered light from a flow field to a molecular absorption cell. The Doppler shift was determined by locking a second laser to a reference point on an identical molecular absorption cell. The frequency difference between the lasers, and thus the velocity of the flow, was determined using standard optical heterodyne detection. Results reported indicate an uncertainty of 15 [m/s] for a 100 ms integration time.

The objective of the current project is to develop a point velocimetry measurement system based on the filtered Doppler velocimetry concept. The system should allow for both mean and unsteady velocity measurements. This goal will be accomplished using a tunable diode laser and a Cesium vapor Faraday cell.

Experiment Setup of the PDV System

The basic PDV system is shown in section 1 of figure 1. A Melles Griot DLD-006 diode laser driver is used to modulate the spectral frequency of light from an SDL DBR diode laser operating at 852 nm. This light is focused into a seeded flow and is Doppler shifted by seed particles in the flow. Scattered light is collected and passed through a Cesium vapor Faraday cell that acts as the frequency discriminator for the system. The intensity of the filtered light is converted to a voltage by a Hamamatsu avalanche photo-diode. The voltage output from the APD is fed to a pair of lock-in amplifiers where the Doppler shift is determined using a harmonic detection scheme. The details of the harmonic detection scheme are covered in appendix A, however, the final equation of interest is:

$$ratio = \frac{I_{L1}}{I_{L2}} = \frac{4\left(\nu_D + \nu_E\right)}{A_h} \tag{1}$$

Here I_{L1} is the voltage output from a lock-in amplifier, which is demodulating the intensity signal from the Faraday cell at the laser modulation frequency. I_{L2} is the voltage output from a lock-in amplifier, which is demodulating the intensity signal from the Faraday cell at twice the laser modulation frequency. A_h is the amplitude of the laser frequency modulation and ν_D is the Doppler shift of the scattered light. ν_E is a term that represents the error between the desired spectral operating frequency, or wavelength, of the laser and the actual spectral operating frequency of the laser. This error term can not be distinguished from a Doppler shift of the scattered light and therefore represents a false velocity signal. To minimize this false velocity signal a feedback loop was created which locks the spectral frequency of the laser to a set location on the transmission curve of the Faraday cell. The components of the control loop are shown in section 2 of figure 1. The design and operation of the control loop is discussed in appendix B.

Experimental Equipment

The current PDV system uses a Faraday cell and a single frequency diode laser as opposed to the more common molecular iodine cell and either Nd:YAG or AR^+ laser. For this reason a short description of the less common components of this PDV system is given.

Faraday Cell:

Much of the early work on filtered Doppler Velocimetry has used an optically thick molecular iodine absorption cell. This cell provided several frequency dependent absorption edges that overlapped the operating wavelength of either an Argon ion or a Nd:YAG laser. The transmission of these absorption edges typically varies from ten to ninety percent over a range of about 500 MHz.

The current FDV system uses a Faraday cell[13 14] as the frequency discriminator. A Faraday cell, shown in figure 2, consists of a magnetooptic atomic vapor placed in an axial magnetic field between a pair of crossed polarizers. The first polarizer linearly polarizes light incident on the cell. When linearly polarized light passes through the atomic vapor in a direction parallel to the magnetic field a rotation of the plane of polarization occurs. This is known as the Faraday effect. Near the atomic resonance lines, the rotation of the plane of polarization shows strong frequency-dependence, this is known as the resonant Faraday effect. The slope and frequency range of the transmission edge of the Faraday cell can be tuned by adjusting the temperature, magnetic field strength, and polarizer angle of the Faraday cell. By setting the frequency range of the linear transmission edge, the system can be tuned to use the full range of the transmission edge over the velocity range of the experiment to be studied. For the current PDV system a Cesium vapor Faraday cell is used. The transmission spectra of the Faraday cell used in the current PDV is shown in figure 3.

Diode Laser and Driver:

The diode laser used in this project is an SDL-5712-H1. The laser is a Distributed Brag Reflector (DBR) which operates at 852 nm and has a spectral bandwidth of less than 3 MHz. The output power of the laser is about 150 mW. The spectral operating frequency of the laser can be tuned thermally as shown in figure 4, or using the laser drive current, also shown in figure 4

The laser is driven by a Melles Griot DLD-006 precision diode laser driver. This driver includes a modulation input that allows the driver current to be modulated by an externally applied voltage.

Results

Results are reported for several experiments. The velocity resolution of the PDV system was tested using a rotating disk as a velocity standard. Next the system was used to make velocity measurements of a seeded jet.

Velocity of a spinning disk

The PDV system was tuned to the cesium transition and the feedback loop was activated to stabilize the laser operating frequency. The integration time of the velocity measurement lock-in amplifiers was set to 1 [s] thus setting the frequency response of the system. A spinning disk was set at the probe location and the velocity of the disk was stepped from 0 to 11 [m/s]. The results of the velocity scan are shown in figure 5. The smallest velocity step, about 0.6 [m/s], is clearly visible. The rms of the velocity signal on each step is less than 0.05 [m/s]. On several of the steps there is a noticeable low frequency drift of the mean velocity. This is believed to be caused by the two-cell scheme of the current system. Locking the laser to one cell and measuring velocity through a second cell assumes that the thermodynamic operating conditions of the two cells are identical. If the thermodynamic conditions of one Faraday cell vary relative to the other Faraday cell the result would be an error in the velocity signal.

Velocity of a seeded jet

The velocity profile of a turbulent jet was measured using the PDV system to demonstrate the PDV systems ability to measure a fluid velocity. A one-half inch jet was set to a velocity of 6.9 [m/s] and seeded with smoke from a theatrical fogger. The PDV system was scanned across the jet profile at a distance of one and a half jet diameter from the nozzle. The velocity profile resulting from a single scan is shown in figure 6. The mean velocity measured through the central region of the jet, 7.4 [m/s], agrees with the velocity calculated based on the stagnation pressure in the jet plenum, 6.9 [m/s], to within 0.5 [m/s]. The velocity approaches 0 [m/s] at the jet boundary but the signal becomes noisy due to the low concentration of seed particles at the jet boundary. The velocity signal contains noise on the order of 1 [m/s], from the time traces of the velocity measurement the frequency of the 1 [m/s] noise appears to be below 1 [Hz].

The PDV system was aligned to the centerline of the jet and the jet velocity was varied from 7.5 to 20.1 [m/s]. A 45-second trace of the velocity measurement was taken at each velocity; these traces are shown in figure 7. Once again, the mean velocity measured by the PDV system, agrees with the velocity calculated based on the stagnation pressure in the jet plenum to within 0.5 [m/s], however, the low frequency noise is still clearly visible. One possible source of the low frequency noise is the previously mentioned drift between the reference Faraday cells and velocity Faraday cells. This could be corrected by eliminating the reference Faraday cell and locking the laser to the Velocity measurement cell. A second possible source of the noise is suggested by figure B3. Although the feedback system does significantly improve the stability of the laser frequency figure B3 indicates that the most significant sources of noise in the closed loop system is have a frequency below 0.5 [Hz]. The low frequency noise evident in the jet velocity measurements is also below 0.5 [Hz]. This suggests improvements in the integrator portion of the feedback loop may be necessary.

Conclusions and Recommendations

A point velocity measurement device has been developed which uses an actively stabilized diode laser and a Cesium vapor Faraday cell to measure the Doppler shift of scattered light. Results indicate an accuracy of 0.5 [m/s] for velocity measurements in a seeded jet and a rms noise level of 0.05 [m/s] on the surface of a spinning disk.

Future work will focus on eliminating the low frequency noise in the system and extending the system frequency response. The elimination of the low frequency noise will be accomplished in two steps. First the reference Faraday cell will be eliminating and the laser will be directly locked to the velocity measurement Faraday cell. Second, the integrator portion of the controller will be redesigned using digital techniques. The frequency response of the system with the improved configuration will then be determined.

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Appendix A: Harmonic Detection of Doppler Shift Using a Faraday Cell

Nomenclature

ν_o	spectral frequency at the well of the Faraday cell stopband.
ν_L	spectral frequency of the laser.
ν_S	spectral frequency of the scattered light.
ν_D	Doppler shift.
ν_E	error signal due to drift between the laser frequency and the Faraday cell stopband.
ν_T	combination of the Doppler shift ν_D, and the error signal ν_E.
f_h	frequency of the laser frequency modulation.
A_h	amplitude of the laser frequency modulation.
$I_{SC}(t)$	scattering intensity.
I_{L1}	output from the lock-in amplifier operating at f_h.
I_{L2}	output from the lock-in amplifier operating at $2f_h$.
G	combined gain of the avalanche photo-diode and lock-in amplifier.

Determining Doppler Shift with a Single Photo-detector

The model for the harmonic detection scheme is shown in figure A1. The transmission of the Faraday cell is modeled as a parabolic well for spectral frequencies near ν_o, the spectral frequency at the center of the Faraday cell stopband. The laser is tuned to spectral frequency ν_o and the spectral frequency of the laser is modulated, or dithered about ν_o with amplitude A_h and frequency f_h. This is accomplished by sending a sinusoidal voltage to the modulation input of the diode laser driver, which results in a modulation of the drive current of the diode laser. There is a linear relationship between diode laser drive current and laser operating spectral frequency as shown in figure 4. Modulation of the diode laser drive current results in a spectral operation frequency for the laser as a function of time of:

$$\nu_L = \nu_o + A_h \sin(2\pi f_h t) \qquad (2)$$

The laser is focused into the flow of interest and scattered light, which has been Doppler shifted by particles in the flow to a spectral frequency of ν_S, is collected.

$$\begin{aligned} \nu_S &= \nu_D + \nu_L \\ \nu_S &= \nu_D + \nu_o + A_h \sin(2\pi f_h t) \end{aligned} \qquad (3)$$

The collected light is passed through, or filtered by, the Faraday cell where the relationship between the spectral frequency of the input light and the intensity of the output light is modeled as

$$I = \left[b + c(\nu_S - \nu_o)^2 \right] I_{SC}(t) \qquad (4)$$

By combining equation (2) and (3) and applying the half angle formula it can be shown that the intensity signal output by the Faraday is

$$I = \left[I_{dc} + 2cA_h \nu_D \sin(2\pi t f_h) - \frac{c A_h^2}{2} \cos(2\pi t (2 f_h)) \right] I_{SC}(t) \qquad (5)$$

The intensity output of the Faraday cell contains three distinct terms, a mean intensity, an intensity that varies at the frequency of the laser modulation, f_h, and an intensity that varies at twice the frequency of the laser modulation $2f_h$. The amplitude of the signal at f_h is a linear function of the Doppler shift times the scattering intensity, the amplitude of the signal at $2f_h$ is a linear function of the scattering intensity.

A pair of lock-in amplifiers are used to demodulate the intensity signal and determine the amplitude of the intensity signal at f_h and $2f_h$. The output of the lock-in amplifier at f_h is

$$I_{L1} = G\left[2cA_h \nu_D \right] I_{SC}(t) \qquad (6a)$$

and the output for the lock-in amplifier at $2f_h$ is

$$I_{L2} = G\left[\frac{cA_h^2}{2} \right] I_{SC}(t) \qquad (6b)$$

A ratio of the lock-in amplifier outputs is

$$ratio = \frac{I_{L1}}{I_{L2}} = \frac{G\left(2cA_h \nu_D\right) I_{SC}(t)}{G \frac{cA_h^2}{2} I_{SC}(t)} = \frac{4\nu_D}{A_h} \qquad (7)$$

Note that the scattering intensity has been eliminated by the ratio. The resulting signal is a function only of the Doppler shift and the amplitude of the modulation of the laser frequency which is known.

Laser Frequency Error Signal

In reality, the spectral frequency of the laser is not exactly ν_o. The actual situation is shown in figure A1, the spectral frequency of the laser ν_l can be written as:

$$\nu_L = \nu_o + \nu_E \qquad (8)$$

Carying this substitution through from equation (2) to equation (7) results in a slight modification of equation (7):

$$ratio = \frac{I_{L1}}{I_{L2}} = \frac{4(\nu_D + \nu_E)}{A_h} \qquad (9)$$

Any drift between the spectral frequency of the laser ν_L , and the stopband of the Faraday cell ν_o, results in a false velocity signal that is proportional to the difference between the desired set point of the laser ν_o and the actual operating point of the laser ν_L. To minimize this false velocity signal a feedback loop was created which locks the spectral frequency of the laser ν_L to the stopband of the Faraday cell ν_o. The details of the control loop are discussed in appendix B.

Appendix B: Analysis of the Laser/Faraday Cell Frequency Locking Loop

From equation (9) it is clear that any difference between the actual spectral operating frequency of the laser and spectral frequency of the well of the stopband of the Faraday cell will result in a false velocity signal. This type false velocity is common to most DGV systems. To minimize the false velocity signal, a feedback loop was created which locks the spectral operating frequency of the laser to the well of the stopband of a Faraday cell.

The details of the laser/Faraday cell locking loop are shown in section 2 of figure 1. The PDV system is operated in its standard way; the spectral frequency of the laser is tuned to the stopband of the Faraday cell. The spectral frequency of the laser is modulated with amplitude A_h at a frequency f_h about the well of the stopband of the Faraday cell. A small portion of the laser beam is deflected and passed through the Faraday cell and onto a photo-diode. Once again, the intensity signal from Faraday cell can be demodulated at the modulation frequency of the laser and twice the modulation frequency of the laser as discussed in appendix A. The result is once again equation (6), however, the deflected portion of the laser beam has not been Doppler shifted, therefore ν_D is zero. An assumption that the intensity of the deflected beam is constant eliminates the need to monitor the scattering intensity and ratio as is required for a seeded flow filed. The result is that demodulating the photo-diode signal at f_h results in a linear function of the difference between the spectral operating frequency of the laser and the spectral frequency at the well of the Faraday cell stopband.

$$I_{L1} = G\left[2cA_h\nu_E\right]I_{SC}(t) \qquad (10)$$

This function is referred to as the error signal. A feedback loop using the error signal was created to lock the spectral operating frequency of the laser to the well of the Faraday cell stopband. With this accomplished, the false velocity signal, which appears in equation (9) of appendix A, should be eliminated.

The feedback loop consists of the DBR laser, Melles Griot diode laser driver, and Cesium vapor Faraday cell previously discussed along with a New Focus photo-diode, an EG&G 5302 Lock-in Amplifier, a summing amplifier, and a proportional and integral controller. The operation of the system is shown in section 2 of figure 1. A sinusoidal signal is taken from the lock-in amplifier and fed through the summing amplifier to the modulation input of the diode laser driver. A small portion of the laser beam is deflected through the Faraday cell and onto the photo-diode where the intensity is converted to voltage. The lock-in amplifier demodulates the voltage output by the photo-diode and the output error signal is fed to the PID controller. The loop is closed when the output of the controller is fed to the summing amplifier where it is combined with the modulation signal from the lock-in amplifier and fed to the diode laser driver.

The design of the control loop was carried out using frequency-based analysis. Each component of the feedback loop was disturbed with sinusoidal inputs of a known frequency and amplitude. The output of each component was monitored for amplitude and phase response. The data was used to create the Bode plots and estimated transfer functions for each component of the feedback loop. The amplitude ratio portion of the Bode plots for several key components of the feedback loop are shown in figure B1. Gains and transfer functions for each component are listed in table B1.

From the information in table B1 it is clear that the 2^{ond} order pole of the lock-in amplifier dominates the frequency characteristics of the system. The system was modeled as a series of simple gains with the lock-in amplifier transfer function. A system gain of 23 [dB] based on the individual transfer compares favorably with the experimentally determined open loop gain of 21 [dB]. The amplitude ratio and phase of the open loop system are shown to be in good agreement with the model transfer function in figure B2. Using this model an analog PID controller was constructed.

The system was operated both open loop and closed loop while the error signal was monitored and the power spectrum of the error signal calculated. The power spectrum voltage was converted to an equivalent velocity assuming 180-degree backscatter velocity detection. The resulting spectra are plotted in figure B3. The feedback loop decreases the noise in the system by a factor of about 10 for all frequencies below 250 Hz.

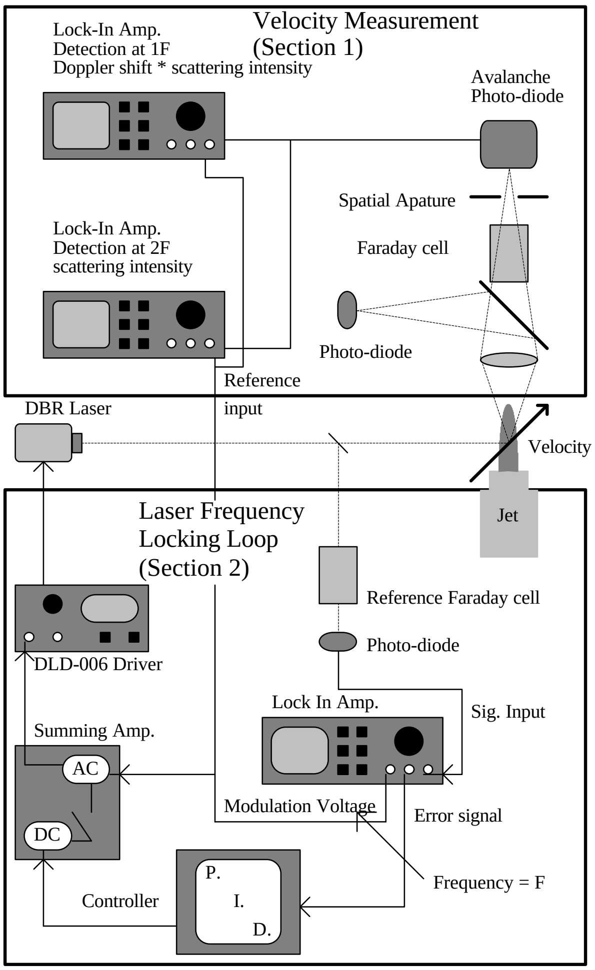

Figure 1 Point Doppler Velocimetry System.

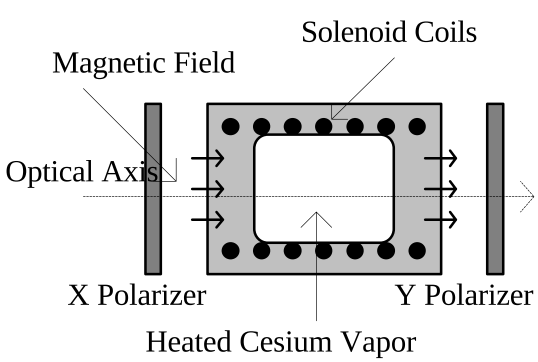

Figure 2 Components of a Faraday Cell.

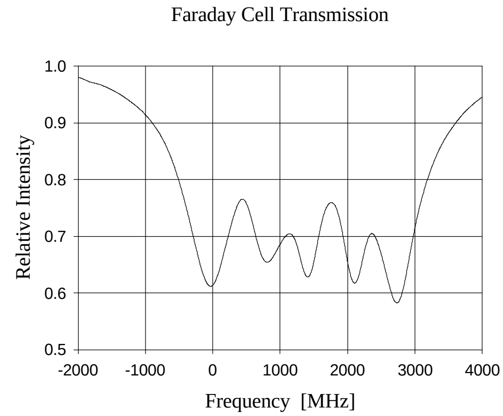

Figure 3 Relative intensity versus frequency for a Faraday cell

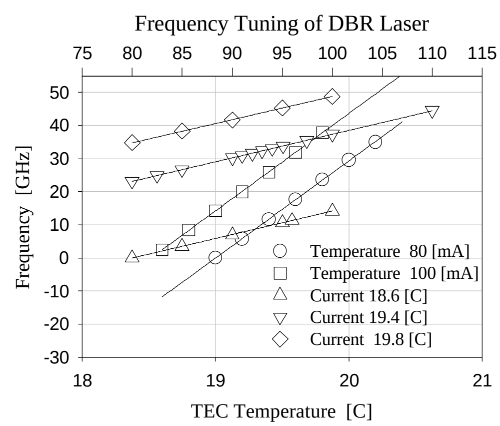

Figure 4 Frequency tuning of the DBR laser using TEC temperature (bottom axis) and operating current (top axis).

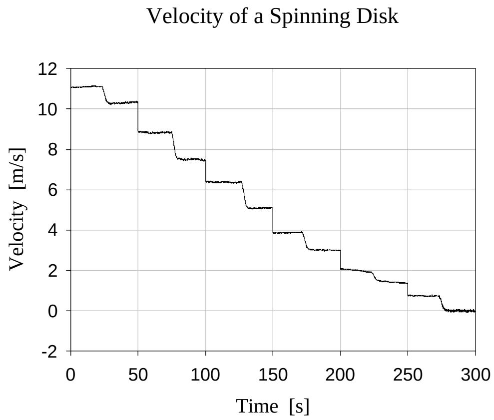

Figure 5 Velocity measurements of a rotating disk using the PDV system. System integration time set to 1 [s].

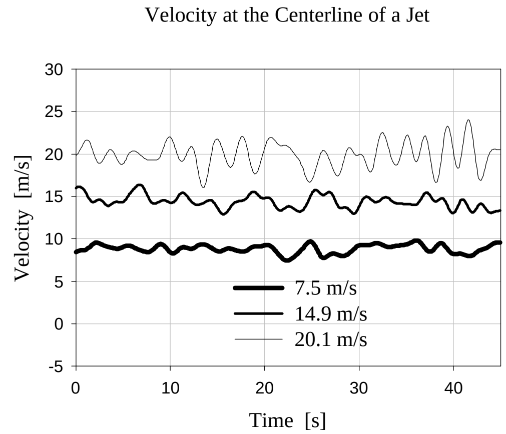

Figure 7 Velocity traces of the jet centerline velocity using the PDV system.

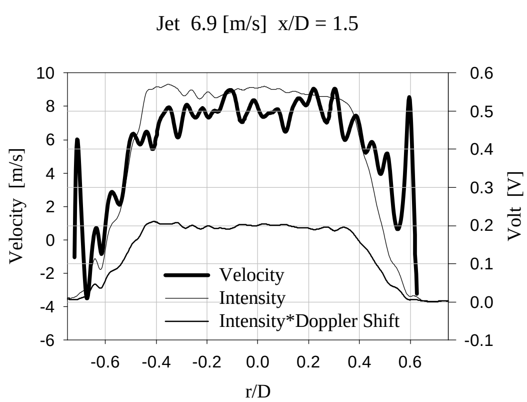

Figure 6 Velocity profile of a jet using the PDV system. System integration time set to 1 [s].

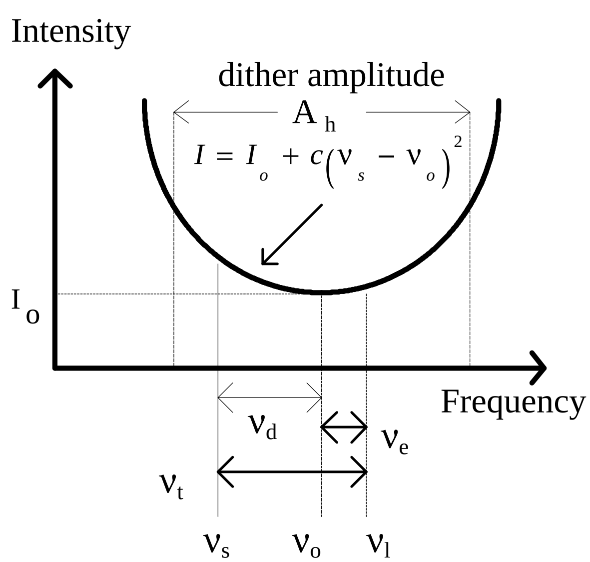

Figure A1 Harmonic detection scheme

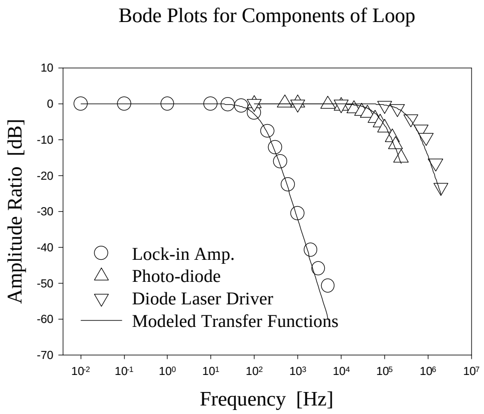

Figure B1 Gain versus frequency for the key components of the Laser Lock loop.

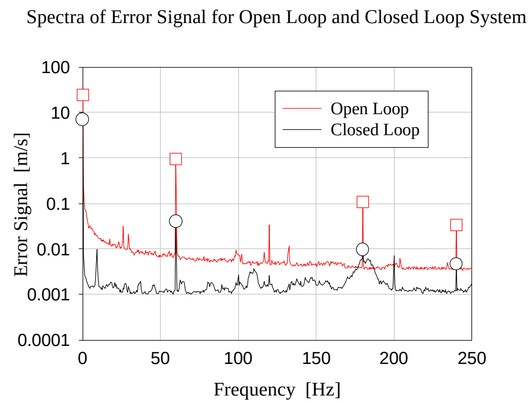

Figure B3 Spectra of the error signal for the open and closed loop systems.

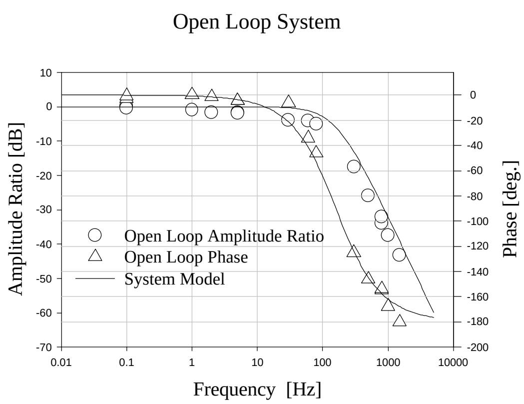

Figure B2 Bode plot of the open loop system compared to the simple model.

Device	Gain	Transfer Function
Sum. Amp. [V]/[V]	-32.0	$SA = \left(\frac{1/628000}{s + 1/628000} \right)$
Lock-in Amp [V]/{V]	30.5	$L_{1ms} = \left(\frac{1/1000}{s + 1/1000} \right)^2$
Driver [mA]/[V]	36.0	$MG = \left(\frac{1/3016000}{s + 1/3016000} \right)^2$
Laser [MHz]/[mA]	57.0	
Photo-diode Faraday Cell [V]/[mW]	-68.5	$PD = \left(\frac{1/628000}{s + 1/628000} \right)^2$

Table B1 Gains and Transfer functions for components of the laser frequency feedback loop.